# COMP 330 Fall 2023
# Assignment 1
# **Due date:** 21$^{\text{st}}$ Sept 2023

Posted on September 5$^{\text{th}}$ 2023

This is the first of 6 assignments. There are **5** questions for credit. The homework is due on `CrowdMark` **at 11:59pm**.

**Important**: Solutions have to be in **pdf** format. Every question must be on a **separate** pdf and submitted through Crowdmark. We prefer solutions prepared on LaTeX but that is **not required**. We will **not** accept jpg photos of handwritten solutions. We will not accept Word documents. Either of these formats can be exported to pdf. **TAs will remove marks for blurry/illegible submissions.**

**Question 1.** [20 points] Prove the following proposition (left as an exercise) from the Lecture 1 notes. Given a set $X$ and an equivalence relation $R$ defined on $X$ and given $x, y \in X$ we have **either** $[x]_R = [y]_R$ **or** $[x]_R \cap [y]_R = \emptyset$ but **not both**.

**Question 2.** [20 points] Fix a finite alphabet $\Sigma$ and let $\emptyset \neq L \subseteq \Sigma^*$. We define the following relation $R$ on strings from $\Sigma^*$:

$$\forall x, y \in \Sigma^*, xRy \text{ if } \forall z \in \Sigma^*, xz \in L \text{ iff } yz \in L.$$

Prove that this is an equivalence relation.

**Question 3.** [15 points] Provide **deterministic** finite automata accepting the following languages over the alphabet $\Sigma = \{a, b\}$.

1. [5 points] $L_1 = \{w \in \Sigma^* : w \text{ contains the substring } aba\}$
2. [5 points] $L_2 = \{w \in \Sigma^* : |w| \geq 1, w \text{ starts with } a \text{ or ends with } b\}$
3. [5 points] $L_3 = \{w \in \Sigma^* : |w| \geq 2, \text{ the second to last letter of } w \text{ is } b\}$

A proof of correctness is not required. No points will be awarded if you provide an NFA.

**Question 4.** [30 points] Let $\Sigma = \{0, 1\}$ and consider the following language

$$L = \{w \in \Sigma^* : w \text{ contains the substring } 10 \text{ or } 01\}$$

1. [10 points] Give a **minimal deterministic** finite automaton $M$ accepting $L$.

2. Provide a “proof sketch” which shows the correctness of your automaton. To do this, use the technique showed in class. That is,

   a. [5 points] Make a claim about your machine’s extended transition function $\delta^*$. **You do not need to prove your claim.**

   b. [5 points] Show, using your claim, that $L(M) = L$.

3. [10 points] Show that your automaton is minimal by showing that no DFA $N$ can exist such that $L(N) = L$ and $N$ has fewer states than $M$.

**Question 5.** [15 points] Provide **non-deterministic** finite automata accepting the following languages over the alphabet $\Sigma = \{a, b\}$. Note: For $w \in \Sigma^*, \sigma \in \Sigma$, we use $n_\sigma(w)$ to mean the number of letters $\sigma$ in $w$.

1. [5 points] $L_1 = \{w \in \Sigma^* : w \text{ contains the substring } ababa\}$

2. [5 points] $L_2 = \{w \in \Sigma^* : n_a(w) \mod 2 = 1 \text{ or } n_b(w) \mod 3 = 2\}$

3. [5 points] $L_3 = \{w \in \Sigma^* : w \text{ starts and ends with the same letter}\}$

A proof of correctness is not required. No points will be awarded for overly complicated automata. In other words, you *must use non-determinism.*