COMP 330 Fall 2023
Assignment 5
Due date: 17th Nov 2023

Posted on November 3rd 2023

This is the fifth of 6 assignments. There are **5** questions for credit. The homework is due on `CrowdMark` **at 11:59pm**.

Important 1: Solutions have to be in **pdf** format. Every question must be on a **separate** pdf and submitted through Crowdmark. We prefer solutions prepared on LaTeX but that is **not required**. We will **not** accept jpg photos of handwritten solutions. We will not accept Word documents. Either of these formats can be exported to pdf. **TAs will remove marks for blurry/illegible submissions.**

Important 2: This assignment, especially Question 2, is *excellent preparation* for the final exam.

Question 1. [15 points] Show that the following language is not context-free using the Pumping lemma for context-free languages.

$$L = \{a^{n_1}b^{n_2}c^n b^{n_3}a^{n_4} : \exists n_1, n_2, n_3, n_4, n \in \mathbb{N} \text{ such that } n_1 = n_3, n_2 = n_4, n \geq 1\}$$

Question 2. [25 points] ***Read this carefully.*** Consider the following languages over the alphabet $\Sigma = \{a, b, c\}$.

$$L_1 = \{a^i b^j c^k : \exists i, j, k \in \mathbb{N} \text{ such that } i < j \text{ and } j > k\}$$

$$L_2 = \{a^i b^{j+n} c^k : \exists i, j, k, n \in \mathbb{N} \text{ such that } i + j < n + k\}$$

For each language decide whether it is

1. Regular.
2. Not regular, but context-free.
3. Not context-free.

If you answer "Regular" for a language, you must provide an FA (DFA, NFA, NFA+ε) which accepts it ***or*** a regular expression which describes it. If you answer "Not regular, but context-free", you must show that the language is not regular using any method/fact seen in class ***and*** provide ***either*** a CFG which generates it ***or*** a PDA (DPDA, NPDA) which accepts it. If you

answer "Not context-free", you must provide a Pumping lemma proof. You do not need to prove the correctness of any of your constructions **but you should explain how they work**.

Question 3. [30 points] Let Σ be a non-empty alphabet and $L \subseteq \Sigma^*$ be a context-free language. Which of the following languages is/are *necessarily* context-free? Note: For some string $w \in \Sigma^*$, w^R is the string reversal.

1. $\texttt{reverse\_1}(L) = \{xy : \exists x, y \in \Sigma^*, x \in L \text{ and } y^R \in L\}$
2. $\texttt{reverse\_2}(L) = \{x : \exists x \in \Sigma^*, x \in L \text{ and } x^R \in L\}$

If a language is *necessarily* context-free, prove your claim. Any construction you make does not require a proof of correctness.

If a language is *not necessarily* context-free, give a counterexample language L which is context-free but for which $\texttt{reverse\_i}(L)$ (`i = 1, 2`) is not context-free. To do so, you must show that your language L is context-free by giving either a PDA which accepts it or a CFG which generates it. You do not need to prove the correctness of your construction but you should explain how it works. In addition, you must give a Pumping lemma proof which shows that $\texttt{reverse}_i(L)$ is not context-free.

Question 4. [15 points] Consider the following language

$$L = \{w\#w : w \in \{a, b\}^*\}$$

Show that this language is decidable by

1. Providing a high-level description of a deterministic semi-infinite tape Turing machine M which decides it.
2. Providing the state transition diagram of M.

You do not need to prove the correctness of your construction.

Question 5. [15 points] Let $\Sigma = \{0\}$ and consider the following function $f : \Sigma^+ \to \Sigma^+$

$$f(x) = x \cdot x \cdot x$$

Show that f is a total computable function by

1. Providing a high-level description of a deterministic semi-infinite tape Turing machine M which computes f.
2. Providing the state transition diagram of M.

You do not need to prove the correctness of your construction.

Supplementary Questions

These questions are not for credit. Do not submit them to `CrowdMark`. Nothing even remotely close to these questions will appear on the final, but they are quite fun so consider trying them in case you finish this assignment early.

Question 6. [0 points] Show that the language $L = \{a^n b^n : n \in \mathbb{N}\} \cup \{a^n b^{2n} : n \geq 0\}$ is context-free but is not accepted by any DPDA.

Question 7. [0 points] Modify your answer to Question 4 to show that the language $L = \{ww : w \in \Sigma^*\}$ is decidable.

Question 8. [0 points] Give the high-level description of a Turing machine which computes $f : \{0\}^+ \to \{0, 1\}$

$$f(x) = \begin{cases} 1 & \text{if } |x| \text{ is prime} \\ 0 & \text{otherwise} \end{cases}$$