$L \subseteq L(G)$

Claim $P(n)$: If $w \in L$ & $|w| = 2n$
Then $w \in L(G)$.

Pf By strong ind on n.

BC $n=0 \Rightarrow |w| = 2\cdot 0 = 0 \Rightarrow w = \varepsilon \quad S \to \varepsilon$ ✓

$n=1 \Rightarrow |w| = 2\cdot 1 = 2 \Rightarrow w = ab$ OR ba

$S \to aSb \to ab$

$S \to bSa \to ba$

IH For some $n \in \mathbb{N}$, $n \geq 1$ assume that $\forall$
$0 \leq k \leq n$, $P(k)$ is true.

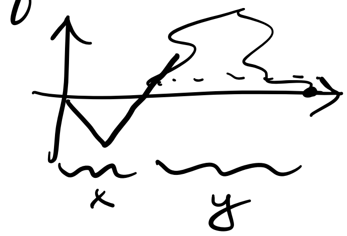
IS Show $P(n+1)$ is true.

Let $w \in L$ s.t. $|w| = 2(n+1) = 2n+2$
$\Rightarrow \quad w = \sigma x \gamma$

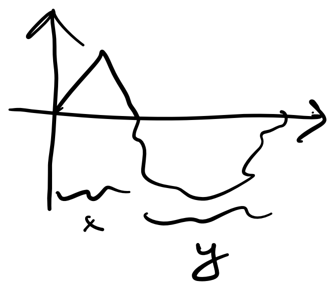
Case 1 $\sigma \neq \gamma$ Then $x \in L$ (why?). Since $|x| = 2n$,
by IH, $S \xrightarrow{*} x$. Then if $\sigma = a$ & $\gamma = b$,
$S \to aSb \xrightarrow{*} axb$. Similarly if $\sigma = b$, $\gamma = a$
with $S \to bSa$

<u>Case 2</u> $\sigma =)$ Then $x \notin L$ (why?) Instead, represent w as $w = xy$

first time cross the x-axis



or



Then $x, y \in L$ & $2 \le |x| \le 2n$ (BC used $n=0$ & $n=1$ because if $n=0$ $2 \le |x| \le 0$ doesn't make sense)

$2 \le |y| \le 2n$

By IH $S \overset{*}{\Rightarrow} x$ & $S \overset{*}{\Rightarrow} y$

$\therefore$ $S \to SS \overset{*}{\Rightarrow} xS \overset{*}{\Rightarrow} xy = w$

$\blacksquare$