

## Post Thm Pf Details for $M$ in $\Leftarrow$

$M :=$  On input  $w$

1. For  $i = 1, 2, 3, \dots$
2. Run  $E_1$  &  $E_2$  for  $i$  steps
3. For each string  $x$  enumerated by  $E_1$ :
4. Check if  $w = x$
5. If yes &  $i = 1$  then accept
6. If yes &  $i = 2$  then reject.

Thm  $\Sigma, \Gamma \neq \emptyset, L_1 \subseteq \Sigma^*, L_2 \subseteq \Gamma^*$   
 $L_1 \leq_m L_2$ , if  $L_2$  is CE then  $L_1$  is CE.

Pf  $L_2$  is CE  $\Rightarrow L_2$  is Turing recognizable

$\therefore \exists$  TM  $M_2$  s.t.  $L(M_2) = L_2$ .

AND  $L_1 \leq_m L_2 \therefore \exists$  mapping reduction  $f$  which is comp.

Design  $M_1$  s.t.  $L(M_1) = L_1$  as follows:

$M_1 :=$  On input  $x$

1. Compute  $f(x)$
2. Run  $M_2$  on  $f(x)$
3. If  $M_2$  accepts  $f(x)$  then accept

$x \in L(M_1) \Leftrightarrow$  if-then-else on 3. evaluates to true

$\Leftrightarrow M_2$  accepts  $f(x)$

$\Leftrightarrow f(x) \in L(M_2) = L_2$

$\Leftrightarrow x \in L_1$  (by  $f$ ).