Theory of Computation

Tutorial - Closure Properties of Regular Languages

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Plan for today

1. Closure properties of regular languages

Closure properties of regular languages

Regular languages

Recall

Definition. A language L is regular if there exists a DFA M that accepts L i.e. such that $L = L(M)$.

Theorem. The set of languages accepted by NFAs is exactly the same as the set of languages accepted by DFAs.

Corollary. A language L is regular if there exists an FA M that accepts it.

If a language L is regular and we apply some language operator to it, does this new language remain regular? If we know that languages L_1, L_2 are regular, can we conclude anything about operations on these languages (i.e. are they regular or not)?

Closure properties of regular language

Definition. If for any regular languages L_1, L_2, $L_1 \Theta L_2$ is regular then we say regular languages are "closed under Θ operation".

Theorem. For any regular languages L_1, L_2 we have the following results:

- $L_1 \cup L_2$ is regular.
- $L_1 \cdot L_2$ is regular.
- L_1^* is regular.
- $\overline{L_1}$ is regular.

Fun exercise: Thinking of operations on languages and proving (or disproving) that they are regular.

Exercise

Exercise. Let L be a regular language. Prove that L^R is regular.

Exercise

Exercise. Let L be a regular language and let $\Sigma = \{a, b\}$. Prove that $\texttt{prefix}(L)$ is regular where $\texttt{prefix}(L) = \{x \in \Sigma^* : \exists y \in \Sigma^*, xy \in L\}$.

Exercise

Exercise. Let L_1, L_2 be two regular languages over $\Sigma = \{a, b\}$. Prove that $\texttt{shuffle}(L_1, L_2)$ is regular where $\texttt{shuffle}(L_1, L_2) = \{x_1 y_1 x_2 y_2 \dots x_k y_k : x_1 x_2 \dots x_k \in L_1, y_1 y_2 \dots y_k \in L_2, x_i, y_i \in \Sigma\}$.

Using closure properties to prove that a language is regular

How can we prove a language L is regular?

1. Show that L is accepted by some FA M. But what if L is made up of many pieces?
2. Identify the sub-languages and language operations $(\cup, \cdot, L^*, \overline{L})$ that make up L and then use closure properties to prove L is regular.

Exercise

Exercise. Let L_1, L_2 be two regular languages. Prove that $L_1 \cap L_2$ is regular.

Exercise

Exercise. Given $L_1 = \{ab^n a : n > 0\}$, $L_2 = \{ba, bba\}$, $L_3 = \{aw : w \in \{a, b\}^*\}$. Is $L_1 \cdot L_2 \cup L_3$ regular?

Exercise

Exercise. Is the language $L = \{a^n b^n : n < 42\}$ regular, why/why not? Give some intuition.

Exercise

Exercise. If L is a regular language, show that language $\texttt{append1}(L) = \{x1 : x \in L\}$ is also regular. ($\Sigma = \{0, 1\}$.)

Exercise

Prove or disprove the following statements.

1. If $L_1 L_2$ is regular and L_1 is finite, then L_2 is regular.
2. If L^R is regular, then $\overline{L}$ is regular.
3. There are no two NON-regular languages L_1, L_2 such that $L_1 \cup L_2$ is regular.