Theory of Computation

Tutorial - Context-Free Grammars and Languages

Cesare Spinoso-Di Piano

Plan for today

1. Context-Free Grammars and Languages
2. Derivation Order
3. Ambiguity

Context-Free Grammars and Languages

Context-Free Grammars

Definition. A **context-free** grammar $G = (V, T, S, P)$ is a grammar where all the production rules have the form

$$A \to x$$

Where $A \in V$ and $x \in (T \cup V)^*$.
Example. Which of the following production rules could be seen in a CFG $G = (\{S, A\}, \{a, b\}, S, P)$?

- $S \to aaA|abS$
- $S \to aAa|aASb$
- $aSa \to SA$

Context-Free Languages

Definition. A language L is **context-free** if and only if there is a context-free grammar G such that $L(G) = L$.

- Are regular languages also context-free languages?
- Are context-free languages also regular?

Example

Example. What language is generated by the following CFG $G = (\{S\}, \{a, b\}, S, P)$ where P is

$$S \to aSa|bSb|\lambda$$

List the generated strings from shortest to longest:

- $S \Rightarrow \lambda$
- $S \Rightarrow aSa \Rightarrow aa$
- $S \Rightarrow bSb \Rightarrow bb$
- $\ldots$
- Any string of the form $w = a_1 a_2 a_3 \ldots a_3 a_2 a_1$. That is w is any palindrome of (even or odd ?) length.

Exercise

Exercise. Which of the following string(s) can be generated by the following grammar $G = (\{S, A, B\}, \{a, b\}, S, P)$, where P is

$$S \rightarrow AaaB | aaB$$
$$A \rightarrow AB$$
$$B \rightarrow aba | S$$

a. aabb
b. aaba
c. aaaba
d. baba

Exercise

Exercise. Give a grammar G that generates the language $L = \{a^n b^k c^l : l = 2k + n, k, l, n \geq 0\}$. Is this language context-free?

Exercise

Exercise. Give a context-free grammar G that generates the language $L = \{a^n b^m : m - n = 1\}$.

Exercise

Exercise. Give a context-free grammar G that generates the language $L = \{uv^R wvu : u, v, w \in \{a, b\}^*, |u| = 2, |w| = 3\}$.

Exercise

Exercise. Give a context-free grammar G that generates the language $L = \{uawb : u, w \in \{a, b\}^*, |u| = |w|\}$.

Derivation Order

Derivation Order

Definition. A derivation sequence is said to be leftmost if in each step the leftmost variable in the sentential form is replaced. If in each step the rightmost variable is replaced, we call the derivation sequence rightmost.

Example

Example. $G = (\{S, A, B\}, \{a, b\}, S, P)$ where P is

$$S \to AB$$
$$A \to aaA|\lambda$$
$$B \to Bb|\lambda$$

The following is a leftmost derivation for *aab*
$S \Rightarrow AB \Rightarrow aaAB \Rightarrow aaB \Rightarrow aaBb \Rightarrow aab$

The following is a rightmost derivation for *aab*
$S \Rightarrow AB \Rightarrow ABb \Rightarrow Ab \Rightarrow aaAb \Rightarrow aab$

Exercise

Exercise. The following grammar generates the language $L(0^*1(0+1)^*)$. Give leftmost and rightmost derivation sequences for the string 00101.

$G = (\{S, A, B\}, \{0, 1\}, S, P)$ where P is

$$S \to A1B$$

$$A \to 0A|\lambda$$

$$B \to 0B|1B|\lambda$$

Ambiguity

Ambiguity

Definition. A grammar G is **ambiguous** if for a string $w \in L(G)$ there exists two **distinct** leftmost (or rightmost) derivation sequences of w.

A visual alternative to using derivation order is using **grammar parse trees**.

Example

Example. The grammar $G = (\{S\}, \{b\}, S, P)$ where P is

$$S \to bS|bb|\lambda$$

is **ambiguous**. Since for $bb \in L(G)$ we have two leftmost (can also consider them rightmost) derivation sequences

$$S \Rightarrow bb$$
$$S \Rightarrow bS \Rightarrow bbS \Rightarrow bb$$

Exercise

Exercise. Show that the following grammar is ambiguous:
$G = (\{S, A, B\}, \{a, b\}, S, P)$ where P is

$$S \rightarrow AB | aaaB$$
$$A \rightarrow a | Aa$$
$$B \rightarrow b$$