Theory of Computation

Tutorial - Minimal DFAs

Cesare Spinoso-Di Piano

Plan for today

1. Minimal DFA

Minimal DFA

Minimal DFA

Given a language L, there are several DFAs $\mathbf{M}$ that can accept it.

Theorem. For every regular language L, there is a unique minimal DFA $\hat{M}$ that accepts it. $\hat{M}$ is minimal in the sense that no other DFA M where $L(M) = L$ has a smaller number of states.

State Reduction Algorithm

The following procedure takes as input any DFA $M = (Q, \Sigma, \delta, q_0, F)$ and outputs an equivalent minimal DFA $\hat{M} = (\hat{Q}, \Sigma, \hat{\delta}, \hat{q_0}, \hat{F})$ (i.e. $L(M) = L(\hat{M})$).

Step 1. Remove all unreachable states from M.
Step 2. Initialize two sets $S_1 \leftarrow Q - F$ and $S_2 \leftarrow F$.
Step $j, (j > 2)$. For each pair $p, q \in S_i$

If $\delta(p, \sigma)$ & $\delta(q, \sigma)$ map to the same set $\forall \sigma \in \Sigma$, then p, q are indistinguishable and stay in the same set they were in Step $j - 1$.

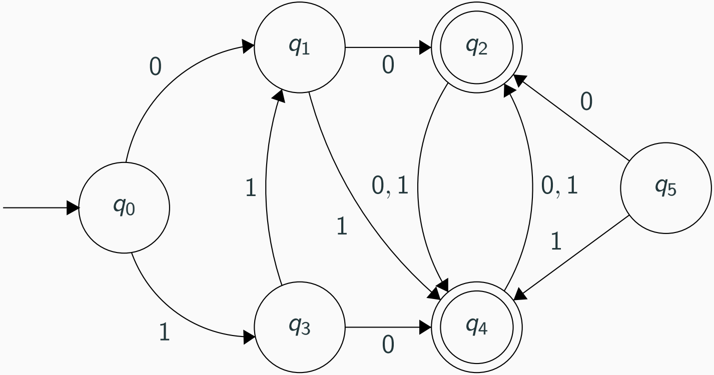
Otherwise, p, q are distinguishable, split the set from Step $j - 1$ into two new sets one with p and another with q. These sets may continue to grow.

If no new sets have been created from $j - 1$ to j, end.
Otherwise, continue.

$\hat{M}$: Each set S becomes a state in $\hat{Q}$. $\hat{q_0}$ is the set S that contains q_0. $\hat{F}$ are the sets that contain at least one final state from F.

Example

Example 1. Reduce the following *DFA* **M**

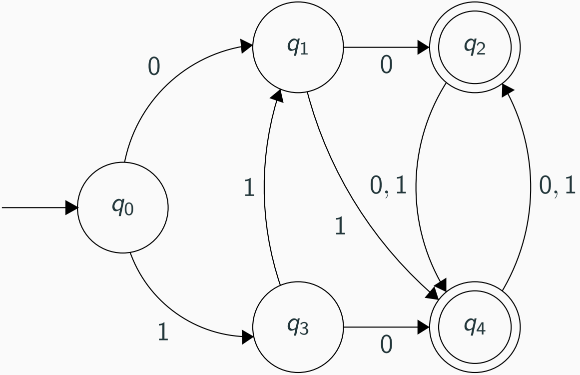


Example

Example 1.

Step 1: Remove all unreachable states from M.

Step 2: Initialize two sets $S_1 \leftarrow \{q_0, q_1, q_3\}$, $S_2 \leftarrow \{q_2, q_4\}$



Example

Example 1.

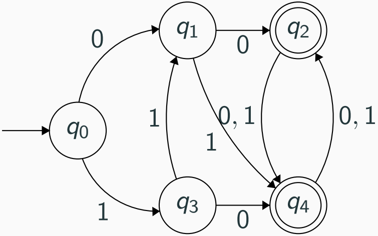
Step j: Distinguishable and indistinguishable states

$\rightarrow \{q_0, q_1, q_3\}, \{q_2, q_4\}$

$\rightarrow \{q_0\}\{q_1\}\{q_3\}\{q_2, q_4\}$

$\rightarrow \{q_0\}\{q_1\}\{q_3\}\{q_2, q_4\}$

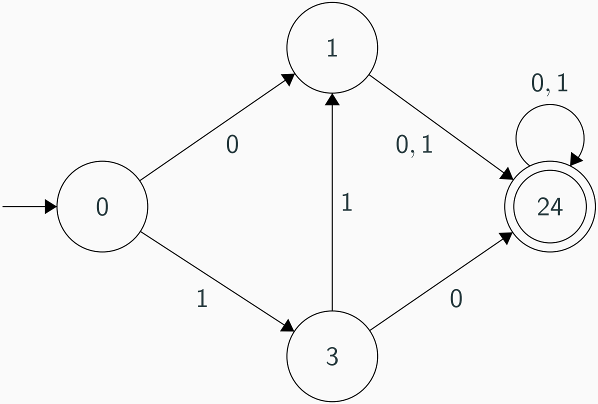
No change from previous step, states have been identified.



Example

Example 1.

Create $\hat{M}$: Each set S becomes a state in $\hat{Q}$. $\hat{q_0}$ is the set S that contains q_0. $\hat{F}$ are the sets that contain at least one final state from F.



Exercise

Exercise 1. Minimize the DFA
$M = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{0, 1\}, \delta, q_0, \{q_2, q_5\})$. Where δ is given as:

δ	0	1
q_0	q_1	q_3
q_1	q_1	q_4
q_2	q_0	q_2
q_3	q_3	q_2
q_4	q_4	q_5
q_5	q_0	q_2